

Operations with Radicals

Simplifying Radicals

1. Use a factor tree to find the prime factorization of the radicand. (Circle each prime factor.)
2. Rewrite the problem, replacing the radicand with its prime factors. (Write the factor in sequential order.)
3. Circle any pairs of identical factors.
4. Cross out each pair of identical factors and pull one of each pair out in front of the radical. Leave unpaired factors under the radical.
5. Multiply where indicated.

$$\sqrt{600x^2y^3}$$

$$\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y}$$

$$\sqrt{\cancel{2 \cdot 2} \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot \cancel{x \cdot x} \cdot \cancel{y \cdot y} \cdot y}$$

$$\sqrt{\cancel{2 \cdot 2} \cdot 2 \cdot 3 \cdot 5 \cdot 5 \cdot \cancel{x \cdot x} \cdot \cancel{y \cdot y} \cdot y}$$

$$2 \cdot 5 \cdot x \cdot y \sqrt{2 \cdot 3 \cdot y}$$

$$10xy\sqrt{6y}$$

Student Practice:

$$\sqrt{480x^8y^7}$$

Quick Method

Look for square root factors and divide variable exponents by the index.

$$\sqrt{250x^2y^3z^4}$$

$$\sqrt{25 \cdot 10 \cdot x^{2/2} \cdot y^{3/2} \cdot z^{4/2}}$$

$$5xyz^2\sqrt{10y}$$

Adding and Subtracting Radicals

1. Square roots with the same radicals can be added or subtracted using the distributive property.
2. In some cases, radicals can be combined once they have been simplified.

Example 1

$$7\sqrt{2} + 9\sqrt{2} = (9 + 7)\sqrt{2} = 16\sqrt{2}$$

Example 2

$$6\sqrt{8} - \sqrt{2}$$

$$6\sqrt{4 \cdot 2} - \sqrt{2}$$

$$6\sqrt{4}\sqrt{2} - \sqrt{2}$$

$$6 \cdot 2\sqrt{2} - \sqrt{2}$$

$$12\sqrt{2} - \sqrt{2}$$

$$(12 - 1)\sqrt{2}$$

$$11\sqrt{2}$$

Student Practice:

$$2\sqrt{20} - 3\sqrt{45} + 3\sqrt{5}$$

- Radial expressions with more than one term are multiplied in much the same way that polynomials with more than one term are multiplied.

Example 1

$$\sqrt{5}(\sqrt{2} + \sqrt{5})$$

$$\sqrt{10} + \sqrt{25}$$

$$\sqrt{10} + 5$$

Student Practice:

$$(\sqrt{2}+7)(6\sqrt{2}+3)$$

Example 2

$$(2 + \sqrt{3})(4 + 5\sqrt{3})$$

$$2 \cdot 4 + 2(5\sqrt{3}) + 4\sqrt{3} + \sqrt{3}(5\sqrt{3})$$

$$8 + 10\sqrt{3} + 4\sqrt{3} + 15$$

$$23 + 14\sqrt{3}$$

Rationalizing the Denominator

The process of rewriting a radical expression as an equivalent expression in which the denominator no longer contains any radicals is called rationalizing the denominator.

- If the denominator contains a square root of a natural number that is not a perfect square, multiply the numerator and the denominator by the radical.

Example 1

$$\frac{4}{\sqrt{7}} = \frac{4}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}}$$

$$\frac{4\sqrt{7}}{7}$$

Student Practice

$$\frac{3}{\sqrt{10}}$$

If the denominator contains two terms, rationalize the denominator by multiplying the numerator and the denominator by the conjugate of the denominator.

$$\frac{9}{7 - \sqrt{5}}$$

$$\frac{9}{7 - \sqrt{5}} \cdot \frac{7 + \sqrt{5}}{7 + \sqrt{5}}$$

$$\frac{9(7 + \sqrt{5})}{7^2 - (\sqrt{5})^2}$$

$$\frac{9(7 + \sqrt{5})}{49 - 5}$$

$$\frac{9(7 + \sqrt{5})}{44}$$

Student Practice

$$\frac{6}{3 + \sqrt{6}}$$