Factoring Polynomials

I. **Is there a greatest common factor**? GCF: shared numerical factor, variables to the smallest exponent, and/or quantity. If factorable, always factor out a negative lead coefficient.

$2x - 8 = 2(x - 4)$

$-x^2y^2 - 3xy^2 = -xy^2(x + 3)$

$5(x - y) + n(x - y) = (x - y)(5 + n)$

II. **Is it a binomial**? If so, is it the…

A. Difference of two squares? parentheses, parentheses, plus and minus—square root both terms

$a^2 - b^2 = (a + b)(a - b)$

$9x^2 - 25y^2 = (3x + 5y)(3x - 5y)$

$(a + b)^2 - 25 = [(a + b) + 5][(a + b) - 5] = (a + b + 5)(a + b - 5)$

B. Sum of two squares? $a^2 + b^2$ does not factor (it is prime.)

C. Sum of two cubes? 1st factor—cube root both terms; 2nd factor—SOPPS (square 1st, opposite, product of terms, plus, square 2nd)

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$8x^3 + 27y^3 = (2x + 3y)(4x^2 - 6xy + 9y^2)$

Note: The resulting trinomial does not factor (it is prime.)

D. Difference of two cubes? 1st factor—cube root both terms; 2nd factor—SOPPS (square 1st, opposite, product of terms, plus, square 2nd)

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$

E. None of these? It does not factor (it is prime)

III. **Is it a trinomial ($ax^2 \pm bx \pm c$)**? If so, is it. . .

A. Square of a binomial (often referred to as a Perfect Square Trinomial)? Are the lead coefficient and c term perfect squares?

$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$

$\mathbf{16}x^2 + 24x \mathbf{+ 9} = (4x + 3)(4x + 3) = (4x + 3)^2$

$\mathbf{4}x^2 - 20xy \mathbf{+ 25}y^2 = (2x - 5y)^2$

B. Is a = 1? Use REVERSE FOIL—read backwards "factors of ***c*** that add (or subtract) to get ***b***; *SIGNS*: if "sum", signs the same; if "difference", signs different and large one gets the sign; now break up the x^2

$x^2 + 7x + 12 = (x + 3)(x + 4)$

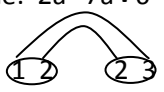
$x^2 - 7x + 12 = (x - 3)(x - 4)$

$x^2 + 3x - 18 = (x + 6)(x - 3)$

$x^2 - 3x - 18 = (x - 6)(x + 3)$

C. Is a $\neq$ 1? Use outer/inner combinations or ***AC GROUPING method***. (See handout on Factoring $Ax^2 + Bx + C$)

Example: $2a^2 - 7a + 6$

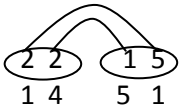


Read in reverse:
1. outer and inner products that **add** to 7
2. SIGNS: if "sum", signs the same
3. break up a^2

3 2
6 1
1 6

$(1x - 2)(2x - 3)$

Example: $4x^2 - 8x - 5$



Read in reverse:
1. outer and inner products that **subtract** to get 8
2. SIGNS: if "difference", signs different and large product (outer/inner) gets the sign
3. break up a^2

$(2x + 1)(2x - 5)$

AC GROUPING method

Example: $2a^2-7a+6$ AC=2 * 6 = 12….factors of 12 that add to get -7

12		-7
12	1	
2	6	
3	4	-3 -4 (would add to -7)

2 3 2 3 (like number-pattern so factor by grouping)

$2a^2 - 3a - 4a + 6$

IV. Does it have four terms. If so, will it. . .

A. Group (first two terms together, last two terms together…*look for a number pattern*)

5/5 5/5 1 1
1 1 1 1

$5ax - 5bx + a - b = (5ax - 5bx) + (a - b)$
$= 5x(a - b) + 1(a - b)$
$= (a - b)(5x + 1)$

1 3 1 3

$x^3 - 3x^2 + 2x - 6 = (x^3 - 3x^2) + (2x - 6)$
$= x^2(x - 3) + 2(x - 3)$
$= (x - 3)(x^2 + 2)$

B. Group (first three terms together)

$x^2 + 6x + 9 - y^2 = (x^2 + 6x + 9) - y^2$
$= (x + 3)^2 - y^2$
$= [(x + 3) + y][(x + 3) - y]$

C. Group (last three terms together)

$y^2 - x^2 + 6x - 9 = y^2 - (x^2 - 6x + 9)$
$= y^2 - (x - 3)^2$
$= [y + (x - 3)][y - (x - 3)]$

BE SURE YOU ANSWER WON'T FACTOR FURTHER.

Summary of the Factoring Process