Properties of Logarithms and Change of Base Theorem

Logarithmic Properties

1. **$\log_a 1=0$**
2. **$\log_a a^x=x$**

    (a white house is a white house)

    likewise $a^{\log_a x}=x$

3. **$\log_a a=1$**
4. **If $\log_a x = \log_a y$ then $x=y$**
5. **$\log_a uv= \log_a u+\log_a v$**
6. **$\log_a u^n=n \log_a u$**
7. **$\log_a(u/v)=\log_a u-\log_a v$**
8. **$\log \sqrt{x} = ½ \log x$**

EXAMPLE: Write the following expression as the sum and/or difference if logarithms. Express all powers as factors.

$$\log \frac{x^2\sqrt{x}}{(x-1)^2}$$

$$\log x^2 + \log x^{\frac{1}{2}} - \log(x-1)^2$$

$$2\log x + \frac{1}{2}\log x - 2\log(x-1)$$

EXAMPLE: Write the following expression as a single logarithm.

$$3\log_a x + \log_a(2x-1) - \log_a(1-x)$$

$$\log_a \frac{x^3(2x-1)}{1-x}$$

PRACTICE:

- Expand using sum, difference and multiples
    - $\log (y/z)$
    - $\log_6 xz^{-3}$
    - $\ln \sqrt[3]{x}$
    - $\ln\frac{(x^2-1)}{x^3}$
    - $\text{Ln} \sqrt{(x^2/y)}$

- Express as a single log:
    - $\log y + \log z$
    - $\log_5 8 – \log_5 t$
    - $(1/3)\log 5x$
    - $\ln x^3 + 2\ln y – 4\ln z$
    - $4[\ln z + \ln (z+5)] – 2 \ln (z-5)$
    - $½[\ln(x+1) + 2 \ln (x-1)] + 3\ln x$

Most Calculators only evaluate logarithmic functions with base 10 or base e. To evaluate logs with other bases, we use the

Change of Base Formula

$$\log_a M = \frac{\log_b M}{\log_b a} = \frac{\log M}{\log a} = \frac{\ln M}{\ln a}$$

EXAMPLE: $\log_7 30 = \frac{\log 30}{\log 7} \approx 1.748$

PRACTICE:

Approximate: $\log_5 63$

Approximate: $\log_4 21$