

## Divisibility Rules

An integer  $N$  is evenly divisible by

- 2 if the last digit is even (0, 2, 4, 6, or 8);  
Example: 123,456 is divisible by 2, because the last digit, 6, is even.
- 3 if the sum of its digits is divisible by 3;  
Example: 123,456 is divisible by 3, because  $1 + 2 + 3 + 4 + 5 + 6 = 21$ , which is divisible by 3. ( $21 \div 3 = 7$ )
- 4 if the number formed by the two last digits is divisible by 4;  
Example: 123,456 is divisible by 4, because 56 is divisible by 4.  
 $(56 \div 4 = 14)$
- 5 if it ends in 5 or 0;  
Examples:
  - 123,456 is not divisible by 5, because the last digit, 6, is not 0 or 5.
  - 12,345 is divisible by 5, because the last digit is 5.
- 6 if it is divisible by both 2 and 3;  
Example: 123,456 is divisible by 6, since it is divisible by both 2 and 3. (See examples for 2 and 3 above.)
- 7 if the number formed after the following two steps is divisible by 7:
  - First, remove the last digit,
  - Then, from the number remaining, subtract two times the digit we just removed;See examples on the back of this page.
- 8 if the number formed by the last three digits is divisible by 8;  
Example: 123,456 is divisible by 8, because 456 is divisible by 8.  
 $(456 \div 8 = 57)$
- 9 if the sum of its digits is divisible by 9;  
Examples:
  - 123,456 is not divisible by 9, since  $1 + 2 + 3 + 4 + 5 + 6 = 21$  is not divisible by 9.
  - 123,453 is divisible by 9, since  $1 + 2 + 3 + 4 + 5 + 3 = 18$  is divisible by 9. ( $18 \div 9 = 2$ )
- 10 if it ends in 0;  
Examples:
  - 123,456 is not divisible by 10, since its last digit, 6, is not 0.
  - 123,450 is divisible by 10, since its last digit is 0.
- 11 for two-digit numbers, if it is double,  
Examples: 11, 22, 33, 44, 55, 66, 77, 88, and 99 are each divisible by 11.  
for larger numbers, if the number formed after the following three steps is divisible by 11:
  - find the sum of alternate digits,
  - find the sum of the remaining digits,
  - find the difference of these two sums;See examples on the back of this page.
- 12 if it is divisible by both 3 and 4.  
Example: 123,456 is divisible by 12, since it is divisible by both 3 and 4.  
(See examples for 3 and 4 above.)

Example: 343 is divisible by 7.

First start with the number	343
Separate the last digit from the rest of the number	34      3
Take two times the last digit	34 $3 \times 2 = 6$
And subtract that answer from the rest of the number	$34 - 6 = 28$

Since this number, 28, is divisible by 7, ( $28 \div 7 = 4$ ), we know that the original number, 343, is divisible by 7.

Example: 12,334 is divisible by 7.

First start with the number	12,334
Separate the last digit from the rest of the number	1233      4
Take two times the last digit	1233 $4 \times 2 = 8$
And subtract that answer from the rest of the number	$1233 - 8 = 1225$

Although it turns out that 1,225 is divisible by 7, ( $1,225 \div 7 = 175$ ), most of us can't do that one in our heads. You can, however, repeat the above steps on this number to see if it is divisible by 7:

First start with the number	1,225
Separate the last digit from the rest of the number	122      5
Take two times the last digit	122 $5 \times 2 = 10$
And subtract that answer from the rest of the number	$122 - 10 = 112$

which you may recognize as divisible by 7, since  $112 \div 7 = 16$ . If not:

First start with the number	112
Separate the last digit from the rest of the number	11      2
Take two times the last digit	11 $2 \times 2 = 4$
And subtract that answer from the rest of the number	$11 - 4 = 7$

which is definitely divisible by 7, so the original number, 123,445 is divisible by 7.

Note: This shows that sometimes you may have to repeat the steps to get a small enough number to know if it is divisible. This can also happen with the rules for 3, 9, and 11.

Example: 623,381 is divisible by 11.

Take the number 623,381.

The alternate digits of **623,381** are 6, 3, and 8, whose sum is  $6 + 3 + 8 = 17$ .  
The remaining digits are 2, 3, and 1, whose sum is  $2 + 3 + 1 = 6$ .  
The difference of these two sums is  $17 - 6 = 11$ , which is divisible by 11,  
so the original number, 623,381, is divisible by 11.