Solving exponential and logarithmic Equations

For a>0 and a≠1, the following properties are true for all x and y for which $\log_a x$ and $\log_a y$ are defined:

One to one properties:

$a^x = a^y$ if and only if x = y

$\log_a x = \log_a y$ if and only if x = y

Inverse properties:

$a^{\log_a x} = x$

$\log_a a^x = x$

Example: Apply basic strategies to solve:

Ex. $(3/4)^x = 64/27$ solve: $4^x = 16$ $7^x = 1/49$ $8^x = 4$

$$\frac{3}{4}^x = (\frac{27}{64})^{-1}$$

$$\frac{3}{4}^x = (\frac{3}{4})^{-3}$$

X = -3

Ex. $\ln e^{5x} = 2$ solve: $\log_4 4^x = 3$ $\log 10^x = -1$

$$\log_e e^{5x} = 2$$

5x = 2

X = 2/5

Undo a fractional power by raising both to a reciprocal power.

Ex: $x^{2/3} = 4$ Remember: when you take a "even" root that there are 2 roots….positive and negative

$$\left(x^{2/3}\right)^{3/2} = \pm(4)^{3/2}$$

$$\left(x^{2/3}\right)^{3/2} = \pm\sqrt{(4)}^3$$

$X = \pm 8$

Solve: $x^{3/4} = 125$

x as an exponent: **Undo powers by taking ln or log of both sides** Taking the log will bring the variable down a level and enable you to solve.

Ex: $e^x - 2 = 21$ Solve: $e^x = 16$ $5e^{2x} = 2$

$$e^x = 23$$

$$\ln e^x = \ln 23$$

$$x = \ln 23 \approx 3.1355$$

$3^{3x-1} = 10$ (use log) $2(3^{2t-5}) - 4 = 1$

Application: How long will it take $1000 to double if compounded continuously at 8.5%?

Use factoring as a tool: factor and apply the zero product property. You will also need to apply a log (ln) to both sides since x is a power

Solve: $e^{2x} + 5e^{x} - 14 = 0$ $e^{2x} + 7e^{x} + 12 = 0$

$$(e^x + 7)(e^x - 2)$$
$$(e^x + 7) = 0 \quad (e^x - 2) = 0$$
$$e^x = -7\ (out\ of\ domain) \quad e^x = 2$$
$$x = \ln 2 \approx 0.693$$

Rewrite as a tool: when in log form rewrite as an exponent to see if the new form is solvable

Solve: ln x = 2 (isolate ln first) 2 ln 3x = 4 3 + 2 ln (2x-1)= 4

$$e^2 = x$$
$$x = e^2 \approx 7.389$$

Apply log properties to simplify before solving: (Note: **Must check** to make sure you are not taking the log of a negative number.)

Ex: $\ln(x - 2) + \ln(2x - 3) = 2 \ln x$ solve: $\ln x + \ln(x - 2) = 1$

$$\ln(x - 2)(2x - 3) = \ln x^2$$
$$(x - 2)(2x - 3) = x^2$$
$$2x^2 - 7x + 6 = x^2$$
$$X^2 - 7x + 6 = 0$$
$$(x - 1)(x - 6) = 0$$

X = 1 and/or x = 6 *chec

(1: out of domain) x = 6

$\ln(x+1)^2 = 2$ $\log(x + 4) - \log x = \log(x + 2)$