

Trigonometric Identities

<p><u>FUNDAMENTAL IDENTITIES</u></p> $\tan A = \frac{\sin A}{\cos A} \quad \cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$ $\sec A = \frac{1}{\cos A} \quad \csc A = \frac{1}{\sin A}$ $\sin(-A) = -\sin A \quad \cos(-A) = \cos A$ $\tan(-A) = -\tan A \quad \cot(-A) = -\cot A$ $\sec(-A) = \sec A \quad \csc(-A) = -\csc A$ $\sin^2 A + \cos^2 A = 1$ $\tan^2 A + 1 = \sec^2 A$ $1 + \cot^2 A = \csc^2 A$	<p><u>SUM AND DIFFERENCE IDENTITIES</u></p> $\sin(A + B) = \sin A \cos B + \cos A \sin B$ $\sin(A - B) = \sin A \cos B - \cos A \sin B$ $\cos(A + B) = \cos A \cos B - \sin A \sin B$ $\cos(A - B) = \cos A \cos B + \sin A \sin B$ $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$
<p><u>DOUBLE-ANGLE IDENTITIES</u></p> $\sin 2A = 2 \sin A \cos A$ $\cos 2A = \cos^2 A - \sin^2 A$ $\cos 2A = 1 - 2 \sin^2 A$ $\cos 2A = 2 \cos^2 A - 1$ $\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$	<p><u>HALF-ANGLE IDENTITIES</u></p> $\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}} \quad \tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}}$ $\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}} \quad \tan \frac{A}{2} = \frac{\sin A}{1 + \cos A}$ $\tan \frac{A}{2} = \frac{1 - \cos A}{\sin A}$
<p><u>COFUNCTION IDENTITIES</u></p> $\sin(90^\circ - A) = \cos A \quad \sin\left(\frac{\pi}{2} - A\right) = \cos A$ $\cos(90^\circ - A) = \sin A \quad \cos\left(\frac{\pi}{2} - A\right) = \sin A$ $\tan(90^\circ - A) = \cot A \quad \tan\left(\frac{\pi}{2} - A\right) = \cot A$ $\cot(90^\circ - A) = \tan A \quad \cot\left(\frac{\pi}{2} - A\right) = \tan A$ $\sec(90^\circ - A) = \csc A \quad \sec\left(\frac{\pi}{2} - A\right) = \csc A$ $\csc(90^\circ - A) = \sec A \quad \csc\left(\frac{\pi}{2} - A\right) = \sec A$	<p><u>SUM-TO-PRODUCT AND PRODUCT-TO-SUM</u></p> $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$ $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$ $\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$ $\sin A + \sin B = 2 \sin\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right)$ $\sin A - \sin B = 2 \cos\left(\frac{A + B}{2}\right) \sin\left(\frac{A - B}{2}\right)$ $\cos A + \cos B = 2 \cos\left(\frac{A + B}{2}\right) \cos\left(\frac{A - B}{2}\right)$ $\cos A - \cos B = -2 \sin\left(\frac{A + B}{2}\right) \sin\left(\frac{A - B}{2}\right)$ $\sin^2 A = \frac{1 - \cos 2A}{2}$ $\cos^2 A = \frac{1 + \cos 2A}{2}$
<p><u>SOLVING TRIANGLES</u></p> <p>Law of Sines:</p> $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ <p>Law of Cosines:</p> $a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$	